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Probabilistic Sizing of Laminates With Uncertainties

A.R. Shah and D.G. Liaw
Sverdrup Technology, Inc.
Lewis Research Center Group
Brook Park, Ohio

and

C.C. Chamis
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio

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A.R. Shah and D.G. Liaw
Sverdrup Technology, Inc.
Lewis Research Center Group
Brook Park, Ohio 44142

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C.C. Chamis
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

ABSTRACT

A reliability based design methodology for laminate sizing and configuration for a special case of composite structures is described. The methodology combines probabilistic composite mechanics with probabilistic structural analysis. The uncertainties of constituent materials (fiber and matrix) to predict macroscopic behavior are simulated using probabilistic theory. Uncertainties in the degradation of composite material properties are included in this design methodology. A multi-factor interaction equation is used to evaluate load and environment dependent degradation of the composite material properties at the micromechanics level. The methodology is integrated into a computer code IPACS (Integrated Probabilistic Assessment of Composite Structures). Versatility of this design approach is demonstrated by performing a multi-level probabilistic analysis to size the laminates for design structural reliability of radome type structures. The results show that laminate configurations can be selected to improve the structural reliability from three failures in 1000, to no

failures in one million. Results also show that the laminates with the highest reliability are the least sensitive to the loading conditions.

1. INTRODUCTION

Design of composite structures is complicated due to the number of variables involved both at the material and structure level. Since different materials are combined at microscopic scale to form the composite, it is complicated to predict the properties of composite at macroscopic level. Also, it is difficult to control the fabrication process of composite structures. Therefore, several parameters involved in making composite structures have variations associated with them.

Traditionally, composite structures are designed based on estimating the lower bound on the material strength and upper bound on the loads. However, the properties of constituents, fabrication process variables, loads and environment have uncertainties associated with them. Also, the behavior of composite material and its strength are dependent on the magnitude of the applied load. Therefore, the material properties and the strength with respect to the stress levels in the composite material need to be considered in the design. Quantifying such effect and its variation is a difficult process. Therefore, the conventional approach yields a design with different safety factors related to different failure modes and hence a non-uniform reliability with respect to different failure modes.

In order to develop a robust design methodology, it is imperative to consider the actual variation in the constituent properties, fabrication process, loads, environment, and material behavior in a rational way. Probabilistic approach provides a logical procedure to arrive at a reliability based design and risk quantification.

During the last decade, NASA Lewis Research Center has been actively engaged in developing a methodology that accounts for design/analysis variables' uncertainties to perform probabilistic assessment of structures. Recently, such a methodology for composite structures has been developed and implemented into IPACS[1] (Integrated Probabilistic Assessment of Composite Structures) computer code. IPACS computer code performs the probabilistic micromechanics and composite structural analysis.

In this paper, a multilevel probabilistic analysis of composite radome structure is performed to size the laminates so as to yield the maximum structural reliability. Uncertainties associated with properties of the constituents, fabrication parameters of a composite material, the structural parameters and the loads are considered in the analysis. IPACS computer code was modified to include the multi-factor interaction equation (MFIE) model[2] to simulate the material degradation. Also, a probabilistic parametric study in relation to the laminate size parameters such as ply thickness, fiber volume ratio, void volume ratio and orientation of fibers is performed to study their effect on the material strength and the structural reliability. A probability based laminate design methodology for a desired level of reliability is outlined and effectiveness of the methodology is demonstrated by probabilistically designing a radome structure of an airframe.

2. IPACS COMPUTER CODE

IPACS is an integrated computer code to perform probabilistic structural analysis of composite structures. Several NASA in-house developed computer codes such as ICAN[3] (Integrated Composite Analyzer), PICAN[4] (Probabilistic Integrated Composite Analyzer), COBSTRAN[5] (Composite Blade Structure Analyzer), and NESSUS[6] (Numerical Evaluation of Stochastic Structures Under Stress) are integrated to develop IPACS computer code. IPACS has many unique capabilities such as preparing the input interactively, performing the probabilistic analysis at different scales of composite, reliability assessment, etc. The modular structure of IPACS allows the user to run each module individually as well as create a data base for the post processing. The details of IPACS code are given in references 1 and 7. However, the details of material degradation model (MFIE) and the flow chart to incorporate it in IPACS are given in the following sections.

3. MATERIAL DEGRADATION MODEL

Due to the variations in the manufacturing processes, the microstructure of a material is never uniform. This nonuniformity means that uncertainties exist in a material's properties. It becomes more predominant in case of composite structures since more than one material forms the composite. Experience has shown that material properties degrade with the type and history of loads[2]. Since, the loads also have uncertainties associated with them, it is important that the uncertainties in the material properties be quantified using probabilistic methods.

A generic material behavior model called the multi-factor interaction equation[2] (MFIE) has been developed at NASA Lewis Research Center. The fundamental assumption for this model is that material behavior can be simulated by primitive variables. The generic form of the equation is:

$$\frac{M_p}{M_{po}} = \prod_{i=1}^{N} \left[\frac{V_f - V}{V_f - V_o} \right]_i^a \qquad (1)$$

where V denotes an effect such as temperature, stress, fatigue cycles, etc., subscript f denotes the value of the effect at final stage when the material property becomes zero, subscript o denotes the value of the effect at reference stage. N is the number of effects included in the model and a is an exponent. M_p denotes the material property. The exponent a can be determined from the experimental data or can be estimated from the anticipated material behavior due to a particular primitive effect. The insufficiency of the experimental data can be compensated by the uncertainties in the exponent.

The above MFIE model has been included in IPACS and the procedural details of incorporating it is depicted in the flow chart shown in Fig. 1. The degradation is performed at the ply level and not the constituent level. Also, the ply property degradation is considered to a function of stress only. Following equation is used to calculate the stress dependent ply properties:

$$\frac{M_p}{M_{po}} = \left(\frac{S_f - \sigma}{S_f}\right)^a \tag{2}$$

where S denotes the strength and σ the stress. Since, the micromechanics theory calculates the strength based on the properties of the constituents, the final strength and the reference properties would be those calculated in the first iteration when the stress level is zero. The exponent in the above equation is assumed to be 0.15 to account for the mild degradation of the polymer matrix composites properties.

As can be seen from the Fig. 1 that the laminate forces are used to check the convergence of stresses which makes the algorithm efficient and fast convergent. The procedure depicted in Fig. 1 is repeated for each perturbation analysis in IPACS to evaluate the sensitivity of random variables.

4. DESIGN OF LAMINATES FOR A RADOME STRUCTURE

A radome structure with 4.27 m diameter hemisphere of an airframe as shown in Fig. 2 is selected to size the laminates of a composite. Generally, the radome structure houses the electrical controls, radars and crew members. The major load on the radome is the air pressure. The distribution of the pressure depends on the shape of a radome. For the design under consideration in this paper a pressure variation from 55.2 Pa at the bottom to 96.5 Pa at the nose has been considered in the analysis. Also, the maximum temperature of 132.2 °F due to aerodynamic heating has been considered in the analysis. The uncertainties in the material properties, ply thickness, fiber volume ratio and the pressure loads are considered in the design process, as these were found to be significant (explained in the next section). Radome is assumed to be made up of graphite/epoxy composite material. The structure is assumed to be fixed at the bottom (where it is attached to the fuselage).

5. DESIGN PROCEDURE

Several different ply configurations and fiber volume ratios given in Table 1 are analyzed probabilistically to assess the reliability of radome. Initially, uncertainties associated with all the constituent material properties, fiber volume ratio, void volume ratio, ply thickness and ply lay up angle were considered in the preliminary analysis. The preliminary analysis results showed that the fiber modules in the longitudinal direction, fiber volume ratio, thickness and pressure loads dominate the reliability. Therefore, in the subsequent detailed analysis with material degradation, only these variables were considered to be random. The uncertainties associated with these variables are listed in Table 2.

Reliability of all these cases are computed and the sensitivity of random variables to the reliability are quantified. The reliability assessment is performed from the strength criterion consideration only. All the strengths namely: tensile, compressive and shear are computed using micromechanics theory[3]. The compressive strength in the longitudinal fiber direction is the minimum of the strengths determined from fiber crushing, delamination and fiber micro-buckling criteria. Following formula is used to compute the reliability:

$$R = 1 - P_f = 1 - \int f_\sigma(X)\, F_S(X)\, dX \qquad (3)$$

where R is the reliability, P_f is the probability of failure, $f_\sigma(X)$ denotes the probability density function of stress and $F_S(X)$ denotes the cumulative probability distribution function of strength.

Reliability of each case is studied together with the respective sensitivity of random variables. Based on this study and considering the realistic situation a best design with the highest reliability (lowest probability of failure) is selected. Numerical results of each case are discussed in the following section.

6. RESULTS AND DISCUSSION

Probabilistic composite structural analysis of a radome structure is performed for the cases listed in Table 1. Actual reliability based design would involve computing reliability for every ply at every node. However, for the demonstration purpose, one of the most critical node (number 253 at the crown, see Fig. 2) was analyzed. Also, the reliability at this node for the zero degree ply was computed to compare results of different cases. It was observed that in all the cases the stress in the transverse direction governed the reliability.

Probability density functions (PDF) of the stress and strength in the transverse fiber direction are computed and plotted for ply configuration $(90/\pm45/0/90)_s$ and FVR of 0.6 (Case 1) and 0.5 (case 2) in Figure 3. The reliability and probability of failure for these cases are given in Table 3. Also, Fig. 3 shows the sensitivity of random variables to the reliability for these cases. The sensitivity of random variables to the ply stress and strength at selected probability levels are given in Table 4.

It is seen from Fig. 3 that for the same ply configuration, the reliability against strength in transverse direction is less when mean FVR is smaller. The plot for sensitivity of random variables (Fig. 3) shows that the uncertainties in the FVR governs the reliability in both the cases. However, the sensitivity of FVR decreases when its mean value reduces. In that case the sensitivities of the uncertainties in other random variables become high.

Fig. 4 shows PDFs of stress and strength and sensitivity of random variables to the reliability for the ply configuration $(0/0/0/90/90)_s$ and FVRs of 0.6 and 0.5 (Cases

(3) and (4) respectively). The reliability and probability of failure for these cases are listed in Table 3 and the sensitivity of random variables to the stress and strength are listed in Table 4. In these cases, conclusion similar to those in cases (1) and (2) can be drawn. However, the reliability for this ply configuration is higher than that in Fig. 3 for their respective FVRs.

PDFs of stress and strength and sensitivity of random variables to the reliability for ply configuration $(0/\pm45/90/0)_s$ and FVR 0.6 and 0.5 (cases (5) and (6) respectively) are plotted in Fig. 5. The reliability and probability of failure for these cases are listed in Table 3 and the sensitivity of random variables to the stress and strength are listed in Table 4. Conclusions for these cases are also similar to those of cases (1) through (4). However, the reliability for this ply configuration is higher than that of the previously described cases.

It can be seen from Table 4 that for all the cases the fiber volume ratio (FVR) and elastic modules of fiber in longitudinal direction controls the scatter of stress, and strength. Therefore, it is obvious that the reliability will also be controlled by these variables which is verified by results plotted in Figures 3 through 5. Also, it shows that the cases with higher FVR have higher reliability. As can be observed from figures 3 through 5 that larger the intersection between the PDFs of stress and strength, the lower the reliability. Hence, to achieve higher reliability, the PDFs of stress and strength should be as far apart as possible. From Table 3, it is quite obvious that cases 3, 5 and 6 shows reliability larger than 0.9999. That means that there is more than one design possible to achieve the reliability greater than 0.9999. However, it is important to judge in a situation like this to pick the best design. Looking at the sensitivity factors against reliability (Figs. 4 and 5) for these three cases, it is obvious that the sensitivity of the pressure load for case (3) is the lowest. Using this important information it can be inferred that case (3) is the best design among cases (3), (5) and (6), since controlling the uncertainties of air pressure (being the natural load) is impossible.

Uncertainties in the FVR and elastic modules can be controlled more easily than those of the loads. Thus, the ply configuration is equally important in determining the best reliable design.

A note must be taken here that the design of the entire structure would involve computing the reliability at every ply of every node against each different design criteria. However, the methodology described in this paper can be used effectively using IPACS to design composite structures.

7. CONCLUSION

The reliability based design methodology for composite structures has been described and demonstrated by probabilistically assessing design concepts of a radome composite structure. The methodology is incorporated in the general purpose computer code IPACS which is used to select possible laminate configuration for a given structural reliability. A multi-factor interaction model to probabilistically assess the effects of material degradation is also included in the assessment. The significance of probabilistic sensitivity factors to the reliability of the design is discussed, as are those of laminate configurations, and uncertainties in constituent properties, fabrication variables, and loads. The methodology generates a wealth of results which provide quantifiable information on which to assess different design with respect to their respective structural reliability. For the composite radome investigated: (1) laminate configurations exist which increase the reliability from 3-failures/one-thousand to; zero-failures/one-million and, (2) the sensitivity of loads decreases with increasing structural reliability.

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Table 1 PLY CONFIGURATION AND FIBER VOLUME RATIO FOR DIFFERENT CASES

Case Number	Ply Configuration	Fiber Volume Ratio
1	$(90/\pm45/0/90)_s$	0.6
2	$(90/\pm45/0/90)_s$	0.5
3	$(0_3/90_2)_s$	0.6
4	$(0_3/90_2)_s$	0.5
5	$(0/\pm45/90/0)_s$	0.6
6	$(0/\pm45/90/0)_s$	0.5

Table 2(a) RANDOM VARIABLES USED IN THE EXAMPLE

Random Variable	Mean Value	Coeff. of Variation	Distribution Type
Fiber Modulus (Ef11)	213.7 GPa	10%	Weibull
Fiber Volume Ratio (fvr)	0.5 or 0.6	10%	Lognormal
Thickness	12.7 mm	5%	Normal
Air Pressure	55.2-96.5 Pa	25%	Normal

Table 2(b): Properties of Graphite/Epoxy Composite Material

Material Properties	Symbol	Unit	Mean
Fiber(AS):			
Normal Modulus (11)	E_{f11}	mpsi	31
Normal Modulus (22)	E_{f22}	mpsi	2
Poisson's Ratio (12)	ν_{f12}	-	0.2
Shear Modulus (12)	G_{f12}	mpsi	2
Thermal Expansion Coef. (11)	α_{f11}	ppm/°F	-0.55
Thermal Expansion Coef. (22)	α_{f22}	ppm/°F	5.6
Tensile Strength	S_{fT}	ksi	400
Compressive Strength	S_{fC}	ksi	400
Matrix (EPOXY):			
Normal Modulus	E_m	mpsi	5
Poisson's Ratio	ν_m	-	0.35
Thermal Expansion Coef.	α_m	ppm/°F	42.8
Tensile Strength	S_{mT}	ksi	15
Compressive Strength	S_{mC}	ksi	35
Shear Strength	S_{mS}	ksi	13
Glass Transition Temp.	T_{gdr}	°F	420

Table 3 QUANTIFIED PROBABILITY OF FAILURE AND RELIABILITY FOR DIFFERENT CASES

Case			Probability of Failure	Reliability
No.	Ply Configuration	FVR		
1	$(90/\pm45/0/90)_s$	.6	0.128E-1	0.987249319
2	$(90/\pm45/0/90)_s$	.5	0.282E+0	0.718075461
3	$(0/0/0/90/90)_s$	.6	0.591E-4	0.999940942
4	$(0/0/0/90/90)_s$	.5	0.292E-2	0.997078100
5	$(0/\pm45/90/0)_s$	.6	0.631E-6	0.999999369
6	$(0/\pm45/90/0)_s$	.5	0.102E-4	0.999989808

Table 4 SENSITIVITY OF STRESS AND STRENGTH TO RANDOM VARIABLES

Case No.		Fiber Modulus		FVR		Thickness		Air Pressure	
		Prob.		Prob.		Prob.		Prob.	
		.001	.999	.001	.999	.001	.999	.001	.999
1	Stress	.33	.10	.92	.99	.05	.00	.20	.03
	Strength	.57	.20	.82	.98	.03	.03	.10	.10
2	Stress	.47	.53	.59	.82	.17	.05	.60	.18
	Strength	.64	.47	.74	.71	.05	.15	.2	.51
3	Stress	.43	.15	.88	.99	.04	.0	.21	.03
	Strength	.75	.27	.66	.96	.02	.02	.07	.10
4	Stress	.58	.39	.75	.92	.06	.01	.30	.05
	Strength	.48	.48	.85	.88	.01	.04	.06	.21
5	Stress	.43	.16	.85	.99	.08	.01	.29	.05
	Strength	.77	.28	.62	.95	.03	.05	.12	.16
6	Stress	.57	.44	.7	.9	.12	.03	.4	.1
	Strength	.54	.49	.84	.81	.03	.09	.11	.32

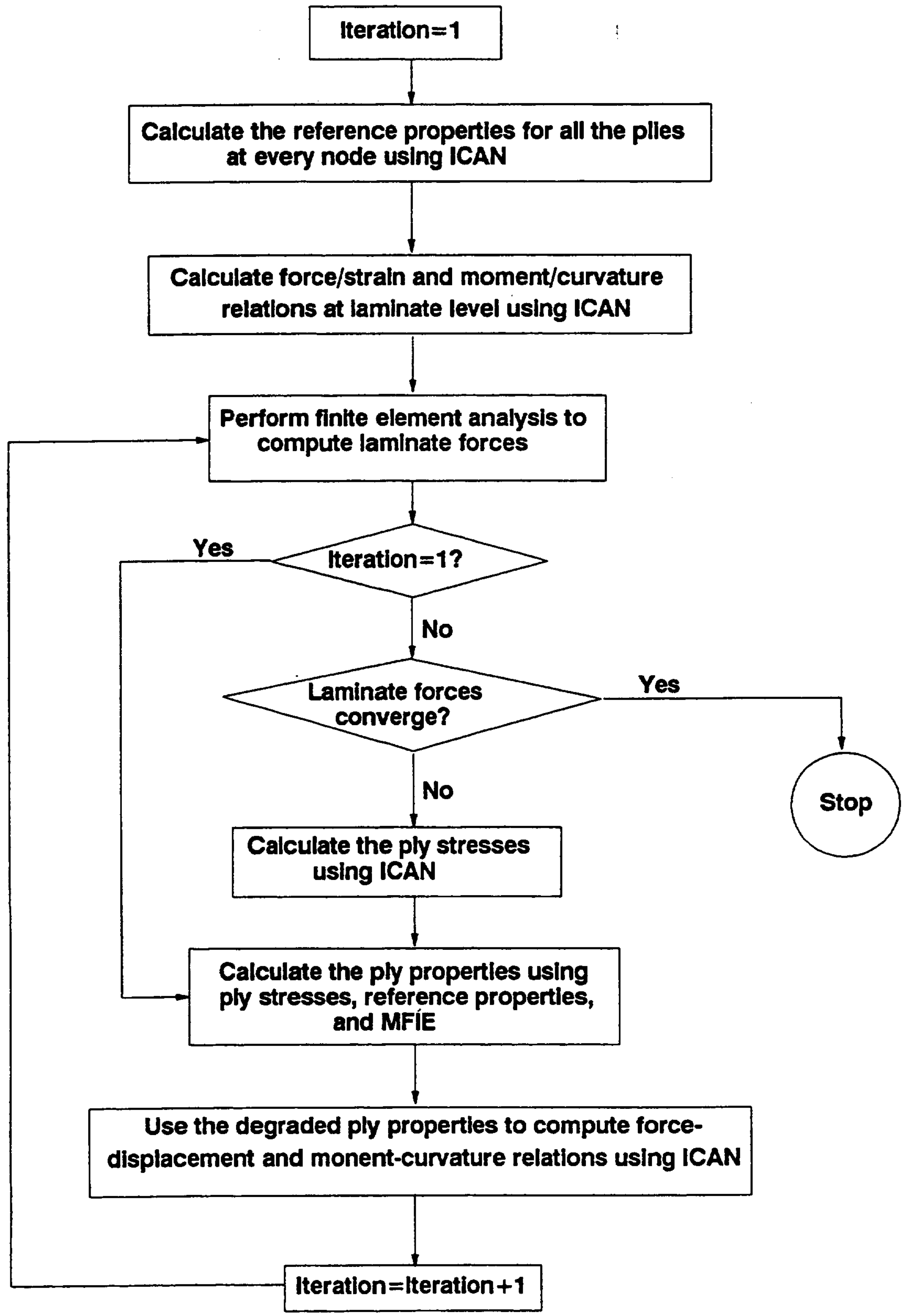


Figure 1.—Flow chart for material degradation assessment.

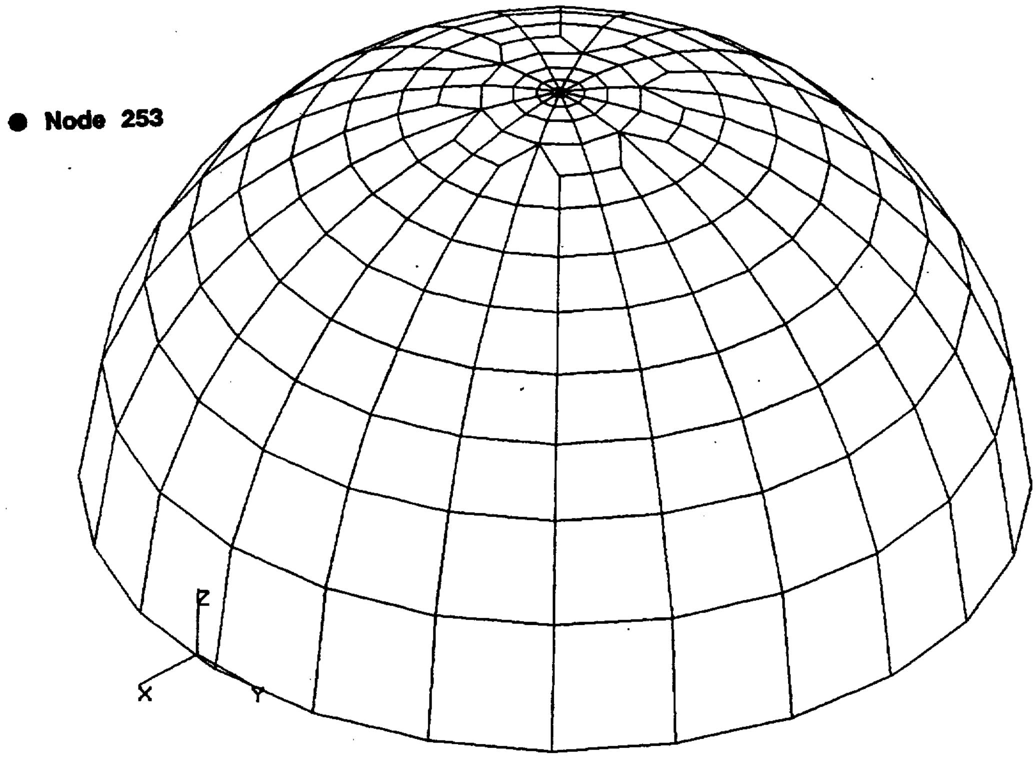


Figure 2.—Radome structure - finite element model.

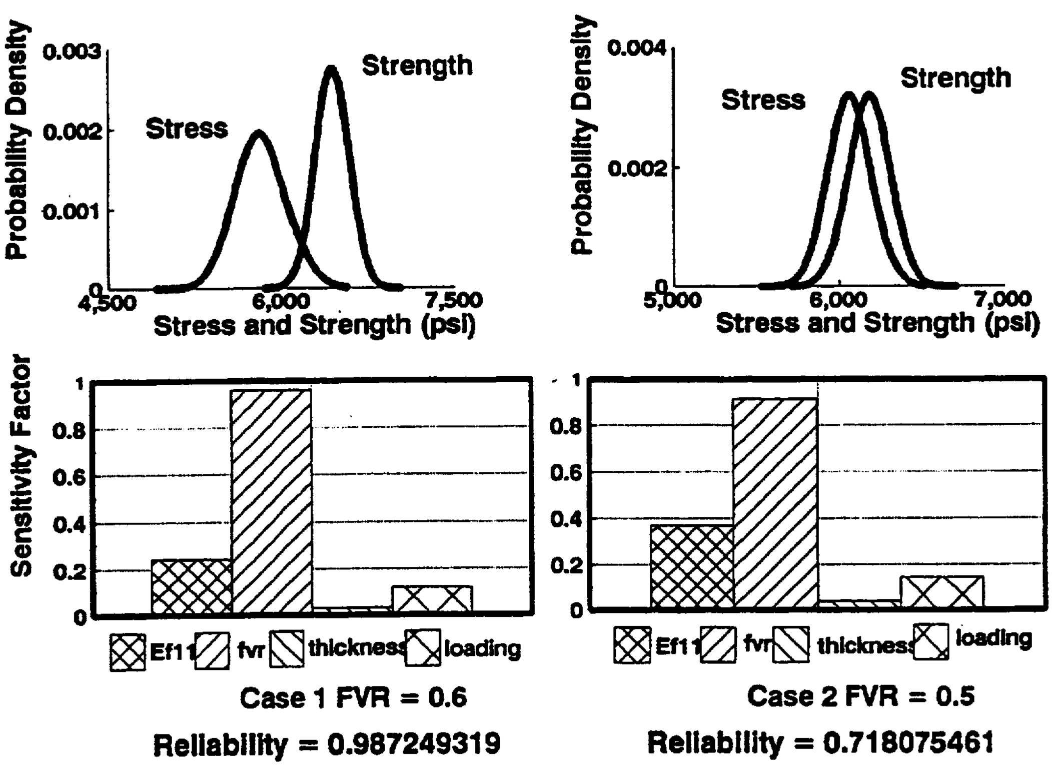
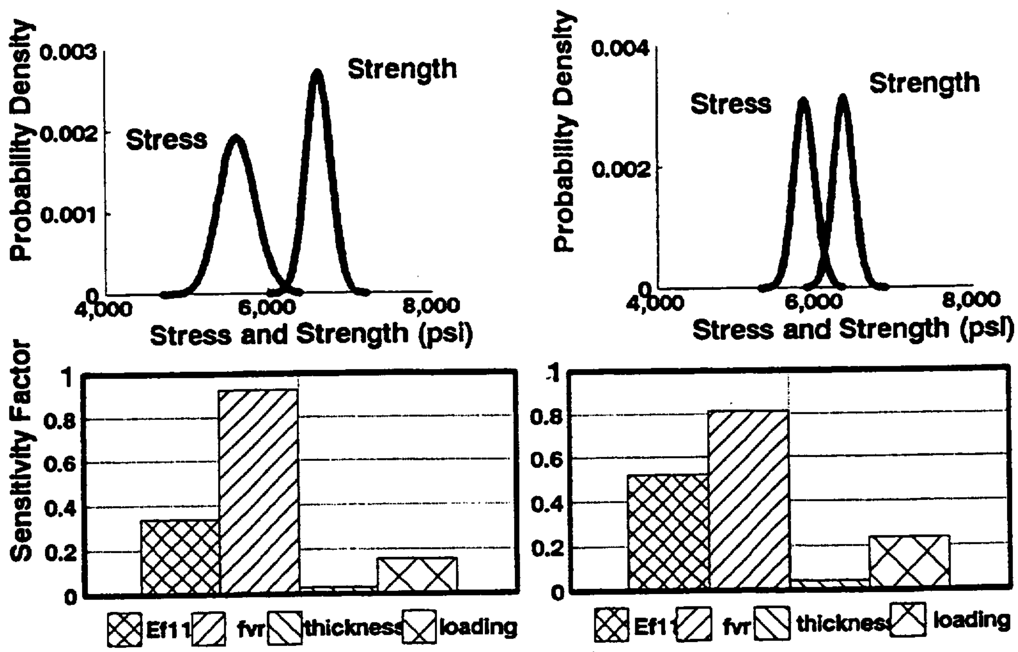


Figure 3.—Reliability analysis of the node at the crown for ply configuration $(90/45/-45/0/90)_s$ and sensitivity of random variables to the reliability.



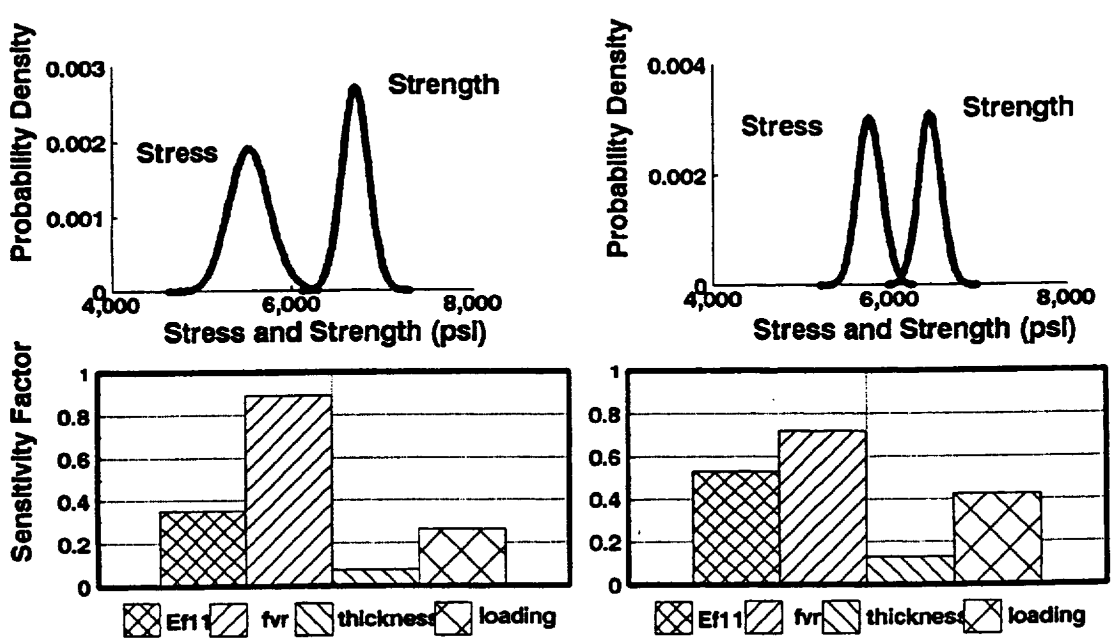
Case 3 FVR = 0.6

Case 4 FVR = 0.5

Reliability = 0.999940942

Reliability = 0.997078100

Figure 4.—Reliability analysis of the node at the crown for ply configuration $(0/0/0/90/90)_s$ and sensitivity of random variables to the reliability.



Case 5 FVR = 0.6

Case 6 FVR = 0.5

Reliability = 0.999999369

Reliability = 0.999989808

Figure 5.—Reliability analysis of the node at the crown for ply configuration $(0/45/-45/90/0)_s$ and sensitivity of random variables to the reliability.

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Lewis Research Center
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13. ABSTRACT ***(Maximum 200 words)***

A reliability based design methodology for laminate sizing and configuration for a special case of composite structures is described. The methodology combines probabilistic composite mechanics with probabilistic structural analysis. The uncertainties of constituent materials (fiber and matrix) to predict macroscopic behavior are simulated using probabilistic theory. Uncertainties in the degradation of composite material properties are included in this design methodology. A multi-factor interaction equation is used to evaluate load and environment dependent degradation of the composite material properties at the micromechanics level. The methodology is integrated into a computer code IPACS (Integrated Probabilistic Assessment of Composite Structures). Versatility of this design approach is demonstrated by performing a multi-level probabilistic analysis to size the laminates for design structural reliability of radome type structures. The results show that laminate configurations can be selected to improve the structural reliability from three failures in 1000, to no failures in one million. Results also show that the laminates with the highest reliability are the least sensitive to the loading conditions.

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